MATH 118: Midterm 2 Key

Name: ______________________________

Directions:

* Show your thought process (commonly said as ”show your work”) when solving each problem for full credit.
* If you do not know how to solve a problem, try your best and/or explain in English what you would do.
* Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
		50

1. Short answer questions:

   (a) Given the function

   $$F(x) = \sqrt{3x^2 - 2}$$

   find two functions f, g where $f \circ g = F$. You are not allowed to choose $f(x) = x$ or $g(x) = x$.

   $\boxed{f(x) = \sqrt{x} \text{ and } g(x) = 3x^2 - 2}$

   (b) Determine whether $x = 2$ is a solution to the equation

   $$\frac{2}{x} - \frac{1}{x-1} = 0$$

   using calculations.

   A few of you solved the equation. To check solutions, just plug in $x = 2$ and see if LHS = RHS.

   $$\text{LHS} = \frac{2}{2} - \frac{1}{2-1} = 1 - \frac{1}{1} = 1 - 1 = 0 = \text{RHS}$$

   $\boxed{x = 2 \text{ is a solution.}}$

   (c) If $x = -3$ is a x-intercept of $P(x)$, what must be a factor of $P(x)$?

   $\boxed{(x - (-3)) = (x + 3)}$

   (d) True or False: The function $g(x) = \sqrt{2x - 2}$ is shifted to the left two units from $f(x) = \sqrt{x}$.

   $\boxed{\text{False.}}$

   We have $g(x) = \sqrt{2x - 2} = \sqrt{2(x-1)}$.

   The correct shift is 1 to the right.

2. Solve the following equations and inequalities:

(a) $-3x - 2 \leq 7$

Don't forget to flip the inequality when dividing by a negative.

$$\begin{aligned} -3x - 2 &\leq 7 \\ -3x &\leq 9 \\ x &\geq -3 \end{aligned}$$

$$\boxed{x \geq -3}$$

(b) $\sqrt{2x - 5} = 4$

Root is isolated. Square both sides and treat like a linear equation.

$$\begin{aligned} \sqrt{2x-5} &= 4 \\ \left(\sqrt{2x-5}\right)^2 &= 4^2 \\ 2x - 5 &= 16 \\ 2x &= 21 \\ x &= \frac{21}{2} \end{aligned}$$

$$\boxed{x = \frac{21}{2}}$$

(c) $\frac{1}{x} - \frac{1}{x-1} = 4$

Multiply both sides by the LCD $x(x-1)$ to rescue x from denominator.

$$\begin{aligned} x(x-1) \cdot \left(\frac{1}{x} - \frac{1}{x-1}\right) &= 4 \cdot x(x-1) \\ \cancel{x}(x-1) \cdot \frac{1}{\cancel{x}} - x\cancel{(x-1)} \cdot \frac{1}{\cancel{x-1}} &= 4x^2 - 4x && \text{Distribute } x(x-1) \text{ and cancel} \\ x - 1 - x &= 4x^2 - 4x \\ 0 &= 4x^2 - 4x + 1 \\ 0 &= (2x-1)^2 && A^2 - 2AB + B^2 \text{ with } A = 2x, B = 1 \\ 0 &= 2x - 1 && \text{Take roots of both sides} \end{aligned}$$

$$\boxed{x = \frac{1}{2}}$$

3. Perform the given instruction.

(a) Determine the end behavior for the polynomial $P(x) = -x^4(2x-3)^2(x-2)^3$.

Find the leading **term**. A few of you picked $-x^4$ which is a **factor**, not a term.

The leading term is $-x^4 \cdot (2x)^2 \cdot x^3 = -x^4 \cdot 4x^2 \cdot x^3 = -4x^9$.

Leading coefficient is $-4 < 0$, with odd degree. So the end behavior is

$$\boxed{y \to -\infty \text{ as } x \to \infty \text{ and } y \to \infty \text{ as } x \to -\infty}$$

(b) Complete the square for the quadratic function $f(x) = 3x^2 - 6x - 5$.

We need $x^2 + bx$ where the coefficient of x^2 is 1. Currently, it is 3. So:

$$
\begin{aligned}
3x^2 - 6x - 5 &= 3(x^2 \overset{b}{-2}x) - 5 \\
&= 3(x^2 - 2x + 1 - 1) - 5 && b = -2 \text{ so } \left(\frac{b}{2}\right)^2 = \left(\frac{-2}{2}\right)^2 = (-1)^2 = 1 \\
&= 3\left[(x^2 - 2x + 1) - 1\right] - 5 \\
&= 3\left[(x-1)^2 - 1\right] - 5 && A^2 - 2AB + B^2 \text{ with } A = x, B = 2 \\
&= 3(x-1)^2 - 3 - 5 && \text{Dist. Law} \\
&= \boxed{3(x-1)^2 - 8}
\end{aligned}
$$

(c) Suppose $g(x) = 1 + \sqrt{-3x - 6}$. Write the order of transformations (including their numbers and direction) you would use to transform $f(x) = \sqrt{x}$ into $g(x)$.

Do not graph.

Put into $A + B \cdot f(C(x+D))$ form by factoring out -3, getting $-3x - 6 = -3(x+2)$.

We have $g(x) = 1 + \sqrt{-3x-6} = \underbrace{1}_{③} + \sqrt{\underbrace{-}_{①}\underbrace{3}_{②}\underbrace{(x+2)}_{④}}$ with transformations

① Reflection around y-axis

② Horizontal shrink by a factor of $\frac{1}{3}$

③ Vertical shift up 1 unit

④ Horizontal shift left 2 units

(d) Find the inverse of the function $f(x) = \dfrac{2x-3}{x-2}$.

You may use the fact that $f(x)$ is one-to-one.

(1) One-to-one. Inverse exists.

(2) Write $y = \dfrac{2x-3}{x-2}$.

(3) Solve for x. Follow 4 steps in Section 1.4.

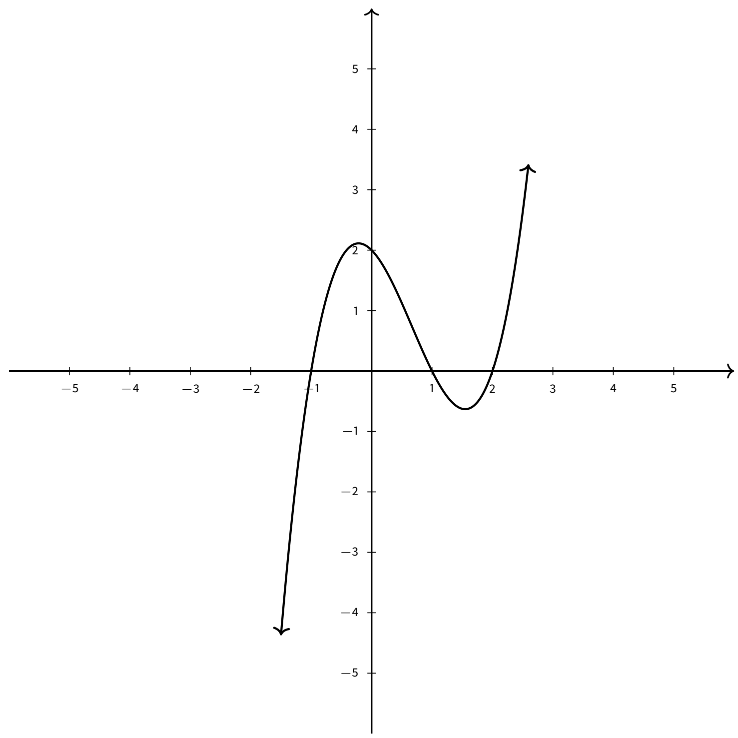
$$\begin{aligned}
y &= \frac{2x-3}{x-2} \\
(x-2)\cdot y &= \frac{2x-3}{\cancel{x-2}} \cdot \cancel{(x-2)} \\
xy - 2y &= 2x - 3 && \text{Global terms} \\
xy - 2x &= 2y - 3 && \text{Get } x \text{ on one side} \\
x(y-2) &= 2y - 3 && \text{Factor } x \text{ out} \\
x &= \frac{2y-3}{y-2} && \text{Isolated}
\end{aligned}$$

(4) Swap. $y = \dfrac{2x-3}{x-2}$.

Result: $\boxed{f^{-1}(x) = \dfrac{2x-3}{x-2}}$

The inverse of $f(x)$ is itself!

4. Suppose $P(x) = x^3 - 2x^2 - x + 2$. Sketch a graph of $P(x)$ using the four step process.



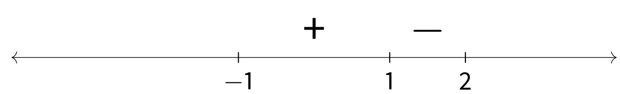
(1) x-intercepts, Solve $P(x) = 0$.

Many of you got this problem incorrect. When solving "polynomial = 0", convert to factors and set each factor to 0. Since we have 4 terms and GCF doesn't work, we have to group.

$$x^3 - 2x^2 - x + 2 = 0$$
$$x^2(x-2) - (x-2) = 0$$
$$(x-2)(x^2-1) = 0$$
$$x - 2 = 0 \qquad x^2 - 1 = 0$$
$$\boxed{x = 2} \qquad x^2 = 1$$
$$\boxed{x = \pm\sqrt{1} = \pm 1}$$

So $x = 3, 1, -1$ are the intercepts.

(2) Test points for sign. Use factored form $P(x) = (x-2)(x^2-1)$ for quick computation of signs.



$P(0) = (0-2)(0^2-1) = - \cdot - = +$

$P(1.5) = (1.5-2)(2^2-1) = - \cdot + = -$

(3) End behavior. Leading term is x^3, odd degree, leading coefficient $1 > 0$. So end behavior is $y \to \infty$ as $x \to \infty$ and $y \to -\infty$ as $x \to -\infty$.

(4) Graph!

5. Perform the given instruction.

(a) Find the domain for each of the following functions:

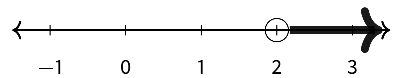
i. $h(x) = \dfrac{1}{\sqrt{x-2}}$

1 Problems.

A. Solve $\sqrt{x-2} = 0$. Squaring both sides, $x - 2 = 0$ so $x = 2$.

B. Solve $x - 2 < 0$. Adding two, $x < 2$.

2 Remove problems $x = 2$ and $x < 2$.



Domain: $\boxed{(2, \infty)}$

ii. $f(x) = \dfrac{1}{x^3 - 9x}$

1 Problems.

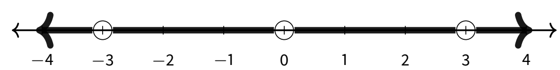
A. Solve $x^3 - 9x = 0$. We have

$$\begin{aligned} x^3 - 9x &= 0 \\ x(x^2 - 9) &= 0 \\ x = 0 \qquad x^2 - 9 &= 0 \\ x^2 &= 9 \\ x &= \pm\sqrt{9} = \pm 3 \end{aligned}$$

so $x = 0, \pm 3$.

B. No root, N/A.

2 Remove problems $x = 0, -3$ and 3.



Domain:

$\boxed{(-\infty, -3) \cup (-3, 0) \cup (0, 3) \cup (3, \infty)}$

(a) Consider $f(x) = -2x + 1$ and $g(x) = x^2 + 2$. Expand **and simplify** the following:

i. $f \circ g$

$$\begin{aligned} (f \circ g)(x) &= f(g(x)) \\ &= f(x^2 + 2) \\ &= -2(x^2 + 2) + 1 \\ &= -2x^2 - 4 + 1 \\ &= \boxed{-2x^2 - 3} \end{aligned}$$

ii. $g(x) - 3f(x)$

$$\begin{aligned} g(x) - 3f(x) &= x^2 + 2 - 3(-2x + 1) \\ &= x^2 + 2 + 6x - 3 \\ &= \boxed{x^2 + 6x - 1} \end{aligned}$$

iii. $f(x)g(x)$ [1]

$$\begin{aligned} f(x)g(x) &= (-2x + 1)(x^2 + 2) \\ &= x^2(-2x + 1) + 2(-2x + 1) \\ &= \boxed{-2x^3 + x^2 - 4x + 2} \end{aligned}$$

[1] This problem should have said to only expand. You got full credit for correct expansion :)